Impacts of Topology and Traffic Pattern on Capacity of Hybrid Wireless Networks

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Abstract—In this paper, we investigate the throughput capacity in wireless hybrid networks with various network topologies and traffic patterns. Specifically, we consider n randomly distributed nodes, out of which there are n source nodes and $n^d(0 < d < 1)$ randomly chosen destination nodes, together with $n^b(0 < b < 1)$ base stations in a network area of $[0, n^w] \times [0, n^{1-w}](0 < w \leq \frac{1}{2})$. We first study the throughput capacity when the base stations are regularly placed and their transmission power is large enough for them to directly transmit to any nodes associated with them. We show that a per-node throughput of $\max\{\min\{n^{b-1}, n^{d-1}\}, \min\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\}\}$ bits/sec is achievable by all nodes. We then investigate the throughput capacity when the base stations are uniformly and randomly placed, and their transmission power is as small as that of the normal nodes. We present that each node can achieve a throughput of $\max\{\min\{\frac{n^{b-1}}{\log n}, n^{d-1}\}, \min\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\}\}$ bits/sec. In both settings, we observe that only when $d > b$ and $d > w$, the maximum achievable throughput can be determined by both the number of base stations and the shape of network area. In all the other cases, the maximum achievable throughput is only constrained by the number of destination nodes. Moreover, the results in these two settings are the same except for the case $d > b > w$, in which the random placement of base stations will cause a degradation factor of $\log n$ on the maximum achievable throughput compared to the regular placement. Finally, we also show that our results actually hold for different power propagation models.

Index Terms—Hybrid wireless networks, throughput capacity, network topology, traffic pattern.

1 Introduction

Capacity has been studied extensively in wireless ad hoc networks. Gupta and Kumar [12] show that the per-node throughput capacity in random ad hoc networks is $\Theta(\frac{1}{\sqrt{n \log n}})$ bits per second, and the per-node transport capacity in arbitrary ad hoc networks is $\Theta(\frac{1}{\sqrt{n}})$ bit-meters per second. Later on, Franceschetti et al. [9] prove by percolation theory that the same $\frac{1}{\sqrt{n}}$ per-node throughput can also be achieved in random ad hoc networks. Recently, Buraagohain et al. [4] study the throughput capacity in grid networks where there are n nodes and the average source-destination distance is d where $d = O(\sqrt{n})$. They show that the $\Omega(1/d)$ per-node throughput can be achieved. The results in [12] can be applied to both *dense* networks where the area is fixed and the density of nodes increases, and *extended* networks where the density of nodes is fixed and the area increases linearly with the number of nodes n, under the assumption that the whole network is connected. Dousse et al. [6] study the throughput capacity in *extended* networks, and find that the throughput cannot be improved much even if a fraction of the nodes are disconnected from the network. In addition, Ozgur et al. [21] also investigate the throughput capacity of a connected ad hoc network. Their results show that by intelligent node cooperation and distributed MIMO communication, the network throughput of dense networks can scale linearly with the number of nodes n, and the network throughput of extended networks can scale as $n^{2-\alpha/2}$ for $2 \leq \alpha < 3$ and $\sqrt{n}$ for $\alpha \geq 3$, where α is the path loss exponent in power propagation model. Moreover, Duarte-Melo et al. [8] study the capacity of *semi-extended* networks, where both node density and the network area increase as the number of nodes n increases. Specifically, they assume the network area is a disk of radius n^γ where $0 < \gamma < \frac{1}{2}$. With a $\frac{1}{(1+d)^\alpha}$ propagation model, they show that the per-node throughput capacity is $\Omega(\frac{1}{n^{1-\gamma}})$, i.e., semi-extended networks cannot scale.

In addition to the above analysis of capacity in static ad hoc networks, there is also a lot of work [3], [10], [11], [16], [19], [20] on the capacity in mobile ad hoc networks. The results show that a constant throughput can be obtained at the cost of very large end-to-end delay under certain mobility models. In this study, in order to possibly improve the network capacity, we place some infrastructures like base stations or access points (APs) into ad hoc networks, which results in the so-called "*Hybrid Wireless Networks*." Here, we only consider the case when nodes are static.

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In the literature, there are some results on the capacity of hybrid wireless networks. Kozat and Tassiulas [14] investigate the throughput capacity of hybrid wireless networks in which both ad hoc nodes and access points are randomly distributed. Their results show that the per-node throughput capacity can be $\Theta(W/\log n)$ bits per second if the number of access points in the network scales linearly with the number of ad hoc nodes, which means the network cannot scale. Similar results are also reported in [1]. Zemlianov and Veciana [25] study the throughput capacity of hybrid wireless networks where ad hoc nodes are randomly distributed and base stations are arbitrarily placed. They show that the per-node throughput capacity depends on the number of base stations, but the network still cannot scale. Considering n randomly distributed nodes and m regularly placed base stations, Liu et al. [17] also study the throughput capacity of hybrid wireless networks. They show that the per-node throughput capacity is $\Theta(\frac{m}{n}W)$ bits/sec if m grows asymptotically faster than $\sqrt{n}$ under k-nearest-cell routing strategy, or if m grows asymptotically faster than $\sqrt{\frac{n}{\log n}}$ under probabilistic routing strategy. Thus, the network can scale if $m = \Omega(n)$. Similar results are also derived in [22], [26]. Recently, Li et al. [15] analyze the throughput capacity in hybrid wireless networks by employing a more efficient resource allocation strategy, and show that the result in [17] is just a special case in their analysis. Besides, they also show that hybrid wireless networks can scale only when $m = \Omega(n)$.

However, we note that all the works above study the throughput capacity of hybrid networks with certain limitations. *First*, they assume the network area is a square, which is not necessarily true in real networks. *Second*, they assume symmetric traffic in the network, i.e., the number of destination nodes is equal to the number of source nodes. While in practice, some of the nodes in the network may be more popular than the others and everyone else wants to communicate to them only, which means the traffic is asymmetric. Liu et al. [18] study the capacity of two-dimensional strip hybrid wireless networks with symmetric traffic. They show that when the width of the strip is at least on the order of the logarithmic of its length, the throughput capacity follows the same scaling law as in the two-dimensional square case. Otherwise, the throughput capacity exhibits the same scaling behavior as in the one-dimensional network case. Toumpis [24] studies the throughput capacity of two-dimensional square ad hoc networks with asymmetric traffic, i.e., when the number of destination nodes is smaller than that of source nodes. They show that if the number of destination nodes is on the order of $\Omega(\sqrt{n})$, a per-node throughput of $\frac{1}{\sqrt{n}}$ can be achieved. Otherwise, the achievable per-node throughput scales linearly with the number of destination nodes.

To serve as a study in a more general setting, this paper investigates the throughput capacity of hybrid networks with the network area to be a rectangle, and the traffic to be asymmetric. As far as we know, this is the first paper in hybrid wireless networks that takes both of these two issues into considerations at the same time.

In this paper, we derive a lower bound on throughput capacity in wireless hybrid networks by presenting an achievable transmission rate. Specifically, we consider n randomly distributed nodes, out of which there are n source nodes and $n^d (0 < d < 1)$ randomly chosen destination nodes, together with $n^b (0 < b < 1)$ base stations in a network area of $[0, n^w] \times [0, n^{1-w}] (0 < w \le \frac{1}{2})$. We first investigate the throughput capacity when the base stations are regularly placed and their transmission power is large enough for them to directly transmit to any nodes associated with them. We show that a throughput of

$$\max\left\{\min\{n^{b-1}, n^{d-1}\}, \min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right\} \text{ bits/sec}$$

is achievable by all nodes. We also investigate the throughput capacity when the base stations are uniformly and randomly placed, and their transmission power is as small as that of the normal nodes. We show that each node can achieve a throughput of

$$\max\left\{\min\left\{\frac{n^{b-1}}{\log n}, n^{d-1}\right\}, \min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right\} \text{ bits/sec.}$$

In both settings, we observe that only when $d > b$ and $d > w$, the maximum achievable throughput is determined by both the number of base stations and the shape of network area. In all the other cases, the maximum achievable throughput is only constrained by the number of destination nodes. Moreover, the results in these two settings are the same except for the case $d > b > w$, in which the random placement of base stations will cause a degradation factor of $\log n$ on the maximum achievable throughput compared to the regular placement. We notice that the results in [14], [18], [24] are just special cases in our results. Besides, we also show that our results actually hold for different power propagation models.

The rest of this paper is organized as follows: In Section 2, we introduce some notations and concepts. Section 3 gives the hybrid wireless network model, including topology model, traffic model, and achievable transmission rate model. In Sections 4 and 5, we derive a lower bound on throughput capacity of hybrid wireless networks, when base stations are regularly and randomly distributed, respectively. After that, we show in Section 6 that our results also hold when we employ another different power propagation model. We finally conclude this paper in Section 7.

2 Notations and Concepts

Here, we introduce some notations and concepts used in this paper.

We use the Knuth's notations [13]:

- $f(n) = O(g(n))$ means $f(n)$ is asymptotically upper bounded by $g(n)$, i.e., $\limsup_{n\to\infty} |\frac{f(n)}{g(n)}| < \infty$.
- $f(n) = \Omega(g(n))$ means $f(n)$ is asymptotically lower bounded by $g(n)$, i.e., $\liminf_{n\to\infty} |\frac{f(n)}{g(n)}| > 0$.
- $f(n) = \Theta(g(n))$ means $f(n)$ is asymptotically tight bounded by $g(n)$, i.e., $0 < \liminf_{n\to\infty} |\frac{f(n)}{g(n)}| \le \limsup_{n\to\infty} |\frac{f(n)}{g(n)}| < \infty$.

Throughput: As defined in the usual way, the time average of the number of bits per second that can be transmitted by each node to its destination is called the *per-node throughput*.

Achievable Throughput: Let $\lambda_i(n)$ denote the throughput of node i. We say that a per-node throughput, denoted by $\lambda(n)$, is *achievable by all nodes* if there exists a spatial and temporal scheduling scheme such that $\lambda_i(n) \geq \lambda(n)$ for all $i \in [1, n]$, and is *achievable on average* if there exists a spatial and temporal scheduling scheme such that $\frac{1}{n}\sum_{i=1}^{n} \lambda_i(n) \geq \lambda(n)$. In this paper, we will derive a per-node throughput achievable by all nodes. Notice that a throughput achievable by all nodes is also achievable on average.

3 Hybrid Wireless Network Model

In this section, we introduce our topology model, traffic model, and achievable transmission rate model for hybrid wireless networks.

3.1 Network Topology

We consider an extended network with n normal nodes and $m = n^b$ $(0 < b < 1)$ base stations in a network area of n. The n nodes are uniformly and independently distributed, while the m base stations are either regularly placed or uniformly and independently distributed, which results in two different kinds of networks that will be discussed in Sections 4 and 5, respectively. Besides, the base stations are interconnected by a wired network, in which the link bandwidth is large enough and there is no bandwidth constraints. Furthermore, we also assume this network area is a rectangle with width $b(n)$ and length $\frac{n}{b(n)}$, where $b(n) = n^w$ and $0 < w \leq \frac{1}{2}$.

3.2 Traffic Pattern

Instead of symmetric traffic mostly assumed in the literature, we assume the network has asymmetric traffic. Specifically, we consider there are n flows in the network. All the n nodes are source nodes while only randomly chosen n^d $(0 < d < 1)$ nodes are destination nodes. Base stations do not serve as data sources or data destinations. Instead, they only help relay packets for the normal nodes.

3.3 Achievable Transmission Rate

Let d_{ij} denote the distance between a node i and another node j. The reception power at node j of the signal from node i, denoted by P_{ij}, follows the power propagation model described in [23], i.e.,

$$P_{ij} = C\frac{P_i}{d_{ij}^{\gamma}}, \tag{1}$$

where P_i is the transmission power of node i, γ is the path loss exponent, and C is a constant related to the antenna profiles of the transmitter and the receiver, wavelength, and so on. As a common assumption, we assume $\gamma > 2$ in outdoor environments [23].

We consider the Shannon Capacity as the achievable transmission rate between two nodes. Specifically, a transmission from node i to node j can achieve transmission rate, R_{ij}, which is calculated as follows:

$$R_{ij} = W\log(1 + SINR_{ij}), \tag{2}$$

where W is the channel bandwidth, and

$$SINR_{ij} = \frac{C\frac{P_i}{d_{ij}^{\gamma}}}{N + \sum_{k \neq i} C\frac{P_k}{d_{kj}^{\gamma}}}$$

is the Signal-to-Interference plus Noise Ratio (SINR) of the signal from node i to node j. In this paper, we assume the n nodes employ the same transmission power $P(n)$ for all their transmissions, i.e., $P_i = P(n)$ for any $i \in [1, n]$. We also consider the channel bandwidth W is a constant.

4 Throughput Capacity with Regularly Placed Base Stations

In this section, we derive a lower bound on the throughput capacity of hybrid wireless networks by presenting an achievable transmission rate. We assume the base stations are regularly placed. Due to the existence of infrastructure, transmissions in the network can be carried out either in infrastructure mode or in ad hoc mode. In the infrastructure mode, packets are first relayed from a source node to the nearest base station, i.e., uplink transmissions, and then carried by the wired network to the base station closest to the destination, and finally forwarded from that base station to the destination node, i.e., downlink transmissions. We evenly divide the bandwidth W into two parts, one for uplink transmissions and the other for downlink transmissions, so that these different kinds of transmissions will not interfere with each other. While in ad hoc mode, packets are forwarded from a source node to a destination node with the help of only normal nodes, i.e., without the help of base stations. Let T_i and T_a denote an achievable per-node throughput by all nodes when all the transmissions are carried out in infrastructure mode and in ad hoc mode, respectively. Then, the maximum per-node throughput achievable by all nodes in hybrid wireless networks, denoted by T, can be calculated as follows:

$$T = \max\{T_i, T_a\}. \tag{3}$$

Since base stations are regularly placed just like in cellular systems, the network is divided into sets of hexagons which are called *cells*. In a cell, a node is closer to the base station in the center than to any other base stations in other cells. We further divide the network into *squares* with length $l = \sqrt{c\log n}$ where $c(c > 1)$ is a constant. Then, we have the following lemma:

Lemma 1. *No square is empty with high probability (w.h.p.).*

Proof. For square i, the probability that there is at least one node in it, denoted by P_i, as $n \to \infty$, is

$$P_i = 1 - \left(1 - \frac{l^2}{n}\right)^n = 1 - e^{n\log(1-l^2/n)} = 1 - \frac{O(1)}{n^c}.$$

So, $P_i \to 1$ as $n \to \infty$. Moreover, let n_s be the number of squares in the network. We have $n_s = \frac{n}{l^2} = \frac{n}{c\log n}$. Then, the probability that every square has at least one node in it, denoted by P_A, is

$$P_A = P_i^{n_s} = \left(1 - \frac{O(1)}{n^c}\right)^{n_s} = e^{-\frac{O(1)}{(cn^{c-1})\cdot\log n}}.$$

Since $c > 1$, we obtain that $P_A \to 1$ as $n \to \infty$, i.e., no square is empty w.h.p. □

Besides, in the network, we allow a transmission between two nodes only when they are located in two

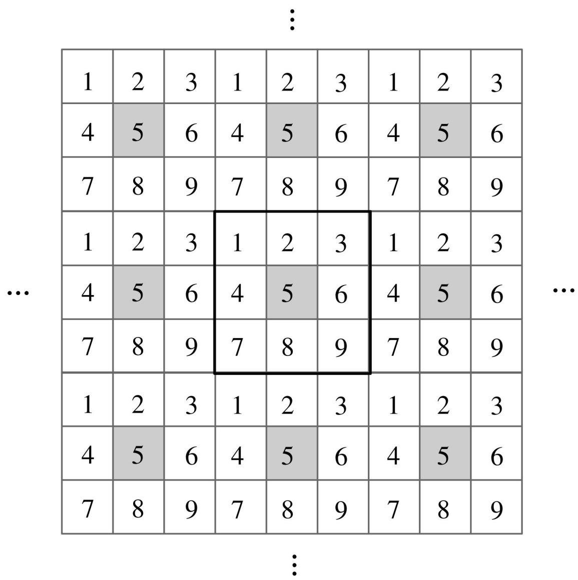

Fig. 1. An example for dividing the network into groups of nine squares.

neighboring squares. Notice that each square can have at most four neighboring squares. Thus, we arrive at the following lemma:

Lemma 2. *In nine time slots, each square in the network has a chance to transmit at a constant transmission rate R, which is independent of* n.

Proof. We divide the network into groups each of which contains nine squares. As shown in Fig. 1, the nine squares in each group are numbered from 1 to 9 in the same way. We further divide time into sequences of successive slots, denoted by t ($t = 0, 1, 2, 3, \dots$). During a slot t, all squares that are numbered $(t \bmod 9) + 1$ are allowed to transmit packets.

Consider a slot when square s_i is allowed to transmit. Then, those squares that may interfere with s_i are located along the perimeters of concentric squares centered at s_i. Since we only allow transmissions between two neighboring squares, at jth tier, there are at most $8j$ interfering squares that are at least $(3j-2)l$ away from the receiver of s_i. Besides, recall that the network is a rectangle with width $b(n)$ and length $\frac{n}{b(n)}$ where $b(n) \leq n^w$ and $0 < w < 1$. Denote the maximum value of j by J. Obviously, we have $j \leq J < +\infty$. Thus, with the power propagation model in (1), the cumulative interference at square s_i, denoted by I_i, can be calculated as

$$\begin{aligned} I_i &\leq \sum_{j=1}^{J} 8j \times \frac{CP(n)}{[(3j-2)l]^\gamma} \\ &\leq \frac{8CP(n)}{l^\gamma}\left[1 + \sum_{j=2}^{J}(3j-2)^{(1-\gamma)}\right] \\ &< \frac{8CP(n)}{l^\gamma}\left[1 + \int_{j=0}^{+\infty}(3j+1)^{(1-\gamma)}dj\right] \\ &< \frac{8CP(n)}{l^\gamma}\left[1 + \frac{1}{3(\gamma-2)}\right] \\ &= \frac{8CP(n)}{l^\gamma} \cdot \frac{3\gamma-5}{3\gamma-6}. \end{aligned} \tag{4}$$

We also need a lower bound on the reception power level at the receiver of s_i, denoted by R_i. Since the maximum distance for a transmitter to a receiver is $\sqrt{5}l$, we can have

$$R_i \geq \frac{CP(n)}{(\sqrt{5}l)^\gamma}. \tag{5}$$

As a result, the SINR at the receiver of s_i, denoted by $SINR_i$, is

$$\begin{aligned} SINR_i &= \frac{R_i}{N_0 + I_i} \\ &\geq \frac{\frac{CP(n)}{(\sqrt{5}l)^\gamma}}{N_0 + \frac{8CP(n)}{l^\gamma} \cdot \frac{3\gamma-5}{3\gamma-6}}, \end{aligned}$$

where N_0 is the ambient noise power at the receiver. By choosing the transmission power $P(n) = c'l^\gamma$ where c' ($0 < c' < +\infty$) is a constant, we can obtain a lower bound on $SINR_i$, i.e.,

$$SINR_i \geq \frac{c'C}{5^{\frac{\gamma}{2}}(N_0 + 8c'C\frac{3\gamma-5}{3\gamma-6})},$$

which is a constant irrespective to the number of nodes n. Thus, referring to (2), we find that a fixed rate transmission rate independent of n can be achieved, which means that in nine time slots, each square in the network has a chance to transmit at a constant transmission rate R. □

Now, we are ready to derive an achievable per-node throughput when all the transmissions are carried out in infrastructure mode, i.e., T_i, and in ad hoc mode, i.e., T_a, respectively.

4.1 Infrastructure Mode Transmissions

As we mentioned before, transmissions in infrastructure mode are carried out in three steps: from a source node to the nearest base station to it, from this base station to the base station nearest to the destination node, and from that base station to the destination node. We analyze the throughput capacity in these three steps, respectively, in the following:

Step 1: from source nodes to base stations.

We assume source nodes have low transmission power and they need to transmit packets to base stations via multiple hops.

Lemma 3. *In each cell, there are at most* $\frac{3n}{m}$ *nodes w.h.p., where* $m = n^b$, *the number of base stations.*

Proof. Let X_i be a random variable denoted as the number of nodes in cell i, and $E[X_i]$ the expectation of X_i. Then, we have $E[X_i] = \frac{n}{m}$.

Recall the Chernoff bounds [5]:

- For any $\delta > 0$,

$$P(X_i > (1+\delta)E[X_i]) < e^{-E[X_i]f(\delta)}, \tag{6}$$

where $f(\delta) = (1+\delta)\log(1+\delta) - \delta$.

- For any $0 < \delta < 1$,

$$P(X_i < (1-\delta)E[X_i]) < e^{-\frac{1}{2}\delta^2 E[X_i]}. \tag{7}$$

Thus, from (6) we can obtain

$$P\left(X_i > \frac{3n}{m}\right) < e^{-\frac{n}{m}\cdot f(2)} = e^{-\frac{c_1 n}{m}},$$

where $c_1 = f(2) = 3\log 3 - 2 > 1$. Since $m = n^b$ and $0 < b < 1$, we have that $P(X_i > \frac{3n}{m}) < e^{-\frac{c_1 n}{m}} \to 0$ as $n \to \infty$.

Besides, the probability that the number of nodes is at most $\frac{3n}{m}$ in all cells, denoted by $P(X_i \le \frac{3n}{m} \forall i)$, can be calculated as

$$P\left(X_i \le \frac{3n}{m}\forall i\right) \ge 1 - mP\left(X_i > \frac{3n}{m}\right) > 1 - me^{-\frac{c_1 n}{m}}.$$

Again, since $m = n^b$ and $0 < b < 1$, we have that $P(X_i \le \frac{3n}{m}\forall i) \to 1$ as $n \to \infty$. □

In Lemma 1, we have shown that in each square, there exists at least one node that can help relay traffic. In Lemma 2, we also show that in nine time slots, each square has a chance to transmit at a constant transmission rate. Besides, by Lemma 3, in each cell, there are at most $\frac{3n}{m} - 1$ nodes that one square has to relay traffic for. Assume each packet has a constant packet size. So, in $O(9 \times \frac{3n}{m})$ time slots, every node is able to transmit a packet toward the base station closest to it.

Denote the throughput capacity in Step 1 by T_{i1}. We can obtain that

$$T_{i1} = \Omega\left(\frac{m}{n}\right) = \Omega(n^{b-1}). \quad (8)$$

Step 2: Infrastructure Relay.

Denote the throughput capacity in Step 2 by T_{i2}. Since we assume base stations are interconnected via a wired network, we can obtain that

$$T_{i2} = \Theta(1). \quad (9)$$

Step 3: from base stations to destination nodes.

We first give two lemmas that will be used later.

Lemma 4. *In each cell, w.h.p., there are at most $2n^{d-b}$ destination nodes when $0 < b < d < 1$, and at most c_3, where $c_3 > \frac{1+b}{b-d}$, destination nodes when $0 < d < b < 1$.*

Proof. Consider cell i. Let Y_i be a random variable denoted as the number of destination nodes in cell i, and $E[Y_i]$ the expectation of Y_i. Then, we have $E[Y_i] = \frac{n^d}{m} = n^{d-b}$.

1. $0 < b < d < 1$.
   According to the Chernoff bound in (6), we can obtain that

   $$P(Y_i > 2n^{d-b}) < e^{-c_2 n^{d-b}},$$

   where $c_2 = f(1) = 2\log 2 - 1 > 0$. Thus, as $n \to \infty$, we have $P(Y_i > 2n^{(d-b)}) \to 0$. Besides, the probability that the number of destination nodes is at most $2n^{d-b}$ in all cells, denoted by $P(Y_i \le 2n^{(d-b)}\forall i)$, can be calculated as

   $$P(Y_i \le 2n^{(d-b)}\forall i) \ge 1 - mP(Y_i > 2n^{(d-b)}) > 1 - n^b e^{-c_2 n^{d-b}},$$

   which approaches to 1 as $n \to \infty$.
2. $0 < d < b < 1$.
   Again, according to the Chernoff bound in (6), we can obtain that

   $$P(Y_i > (1+\delta)E[Y_i]) < e^{-E[Y_i][(1+\delta)\log(1+\delta)-\delta]} = \frac{e^{\delta E[Y_i]}}{(1+\delta)^{(1+\delta)E[Y_i]}}.$$

   Let $1 + \delta = \frac{c_3}{E[Y_i]} = c_3 n^{b-d}$ where c_3 is a constant that will be determined later. Then, we have

   $$\begin{aligned} P(Y_i > c_3) &< \frac{e^{(c_3 n^{b-d}-1)n^{d-b}}}{(c_3 n^{b-d})^{c_3}} \\ &= \frac{e^{c_3 - n^{d-b}}}{c_3^{c_3} n^{c_3(b-d)}} \\ &< \frac{e^{c_3}}{c_3^{c_3}} \cdot n^{c_3(d-b)} \to 0, \text{ as } n \to \infty. \end{aligned}$$

   Besides, we can also obtain that

   $$\begin{aligned} P(Y_i \le c_3 \forall i) &\ge 1 - mP(Y_i > c_3) \\ &> 1 - n^b \frac{e^{c_3}}{c_3^{c_3}} \cdot n^{c_3(d-b)} \\ &= 1 - \frac{e^{c_3}}{c_3^{c_3}} \cdot n^{c_3 d - (c_3-1)b}. \end{aligned}$$

   When we choose $c_3 > \frac{1+b}{b-d}$, we can get $c_3 d - (c_3 - 1)b < -1$, and hence, $P(Y_i \le c_3 \forall i) \to 1$, as $n \to \infty$. □

Lemma 5. *For each destination node, w.h.p., there are at most $2n^{1-d}$ source nodes destined to it.*

Proof. Consider destination node i. Let N_i be a random variable denoted as the number of source nodes that have i as their destination node, and $E[N_i]$ the expectation of N_i. Then, we have $E[N_i] = n \cdot \frac{1}{n^d} = n^{1-d}$.

According to the Chernoff bound in (6), we can obtain that

$$P(N_i > 2n^{1-d}) < e^{-c_2 n^{1-d}},$$

where $c_2 > 0$ and $1 - d > 0$. So, $P(N_i > 2n^{1-d}) \to 0$ as $n \to \infty$. Besides, we also have

$$\begin{aligned} P(N_i \le 2n^{1-d}\forall i) &\ge 1 - n^d P(N_i > 2n^{1-d}) \\ &> 1 - n^d e^{-c_2 n^{1-d}}, \end{aligned}$$

which approaches to 1 as $n \to \infty$. □

As mentioned before, we assume the transmission power of base stations is strong enough so that base stations can directly transmit to destination nodes within the cells. Besides, as in cellular systems, we use seven-cell frequency reuse to enable adjacent cells to transmit at the same time with no interference. Then, it is easy to show that downlink transmissions can also have a constant transmission rate. In addition, from Lemmas 4 and 5, we find that when $0 < b < d < 1$, in each cell, w.h.p., the number of flows from base station to destination nodes is at most $2n^{d-b} \times 2n^{1-d}$, i.e., $4n^{1-b}$, and when $0 < d < b < 1$, in each cell, w.h.p., the number of flows from base station to

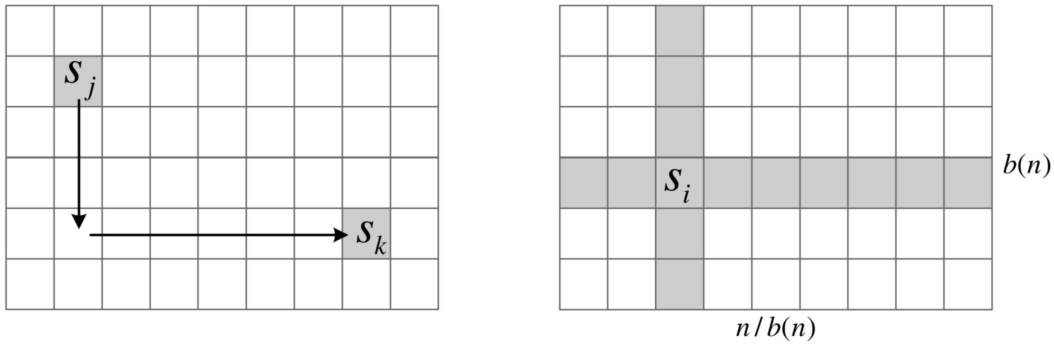

Fig. 2. The routing protocol used for ad hoc mode transmissions.

destination nodes is at most $c_3 \times 2n^{1-d}$, i.e., $2c_3n^{1-d}$. Denote the throughput capacity in Step 3 by T_{i3}. Thus, we can obtain that

$$T_{i3} = \begin{cases} \Omega(n^{b-1}), & \text{when } 0 < b < d < 1, \\ \Omega(n^{d-1}), & \text{when } 0 < d < b < 1. \end{cases} \quad (10)$$

Notice that the minimum of T_{i1}, T_{i2}, and T_{i3} is an achievable throughput for transmissions in infrastructure mode. Thus, combining (8)-(10), we can obtain

$$T_i = \Omega(\min\{T_{i1}, T_{i2}, T_{i3}\}) = \Omega(\min\{n^{b-1}, n^{d-1}\}). \quad (11)$$

4.2 Ad Hoc Mode Transmissions

Transmissions in ad hoc mode are carried out only with the help of normal nodes. We employ the following routing strategy to relay the packets. Specifically, as shown in Fig. 2a, assume a source node is located in square s_j and its destination node is located in square s_k. Packets from this source node are first relayed along those squares that have the same x-coordinate as square s_j until they arrive at a square that has the same y-coordinate as square s_k. Then, these packets are relayed along the squares that have the same y-coordinate as square s_k until they arrive at the destination node.

Consider an arbitrary square s_i in the network, as shown in Fig. 2b. Let N_{i1} and N_{i2} denote the number of source nodes which are located in squares with the same x-coordinate as s_i, and the number of destination nodes which are located in squares with the same y-coordinate as s_i, respectively. Thus, we have

$$E[N_{i1}] = n \cdot \frac{l}{n/b(n)} = l \cdot b(n) = n^w\sqrt{c\log n},$$
$$E[N_{i2}] = n^d \cdot \frac{l}{b(n)} = n^{d-w}\sqrt{c\log n}.$$

Along the line in Lemma 4, we can obtain the following lemma. The proof is included in Appendix A.

Lemma 6. *For all squares, w.h.p.,*

1. *There are at most $2n^w(c\log n)^{\frac{1}{2}}$ source nodes which are located in squares with the same x-coordinate.*
2. *The number of destination nodes which are located in squares with the same y-coordinate is at most $2n^{d-w}(c\log n)^{\frac{1}{2}}$ when $0 < w < d < 1$, and at most c_4 when $0 < d < w \le \frac{1}{2}$, where c_4 is a constant and $c_4 > \frac{2}{w-d}$.*

Denote the number of flows that will cross square s_i as F_i. Since each source node only generates one flow, and there are at most $2n^{1-d}$ flows for each destination node as shown in Lemma 5, we can obtain that for all i,

$$F_i \le N_{i1} + 2n^{1-d}N_{i2}$$
$$\le \begin{cases} 2n^w(c\log n)^{\frac{1}{2}} + 4n^{1-w}(c\log n)^{\frac{1}{2}}, & \text{when } 0 < w < d < 1, \\ 2n^w(c\log n)^{\frac{1}{2}} + 2c_4n^{1-d}, & \text{when } 0 < d < w < 1, \end{cases}$$

i.e.,

$$F_i \le \max\{n^w(\log n)^{\frac{1}{2}}, n^{1-w}(\log n)^{\frac{1}{2}}, n^{1-d}\}. \quad (12)$$

Recall that we have proved in Lemma 2 that a constant rate can be achieved for each transmission. Thus, from (12), we can have that

$$T_a = \Omega\left(\min\left\{\frac{n^{-w}}{\sqrt{\log n}}, \frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right).$$

Since $w \le \frac{1}{2}$, we get $-w \ge -\frac{1}{2}$ and $w - 1 \le -\frac{1}{2}$. So, we obtain that

$$T_a = \Omega\left(\min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right). \quad (13)$$

4.3 An Achievable Throughput

Substituting the results in (11) and (13) into (3), we have

$$T = \max\left\{\Omega(\min\{n^{b-1}, n^{d-1}\}), \Omega\left(\min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right)\right\}$$
$$= \Omega\left(\max\left\{\min\{n^{b-1}, n^{d-1}\}, \min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right\}\right). \quad (14)$$

- Case 1: $w = \frac{1}{2}, d \to 1, 0 < b < 1$.

In this case, the network area is a square, and the traffic in the network is symmetric. Thus, (14) changes into

$$T = \Omega(\max\{n^{b-1}, (n\log n)^{-\frac{1}{2}}\}),$$

which is the same as the result in [24], [18]. It reveals that when we have a square network with symmetric traffic, the throughput capacity is determined by the number of base stations deployed in the network. Specifically, when

$m = n^b = O(\sqrt{n})$, ad hoc mode transmissions can have a higher data rate and we should allow more transmissions in ad hoc mode than those in infrastructure mode to achieve higher throughput. The maximum achievable throughput can be $\Omega(\frac{1}{\sqrt{n \log n}})$ when we carry out all the transmissions in ad hoc mode. However, when $m = n^b = \Omega(\sqrt{n})$, infrastructure mode transmissions can contribute more to the throughput, and we should prefer transmissions in infrastructure mode to obtain higher throughput. Accordingly, the maximum achievable throughput can be $\Omega(n^{b-1})$, i.e., proportional to the number of base stations in the network, when we carry out all the transmissions in infrastructure mode.

- Case 2: $w = \frac{1}{2}, b \to 1, 0 < d < 1$.

This is another extreme case where the network area is a square, the traffic in the network is asymmetric, and there are a lot of base stations, precisely speaking, on the order of n. From (14), we have that

$$T = \Omega(\max\{n^{d-1}, \min\{(n \log n)^{-\frac{1}{2}}, n^{d-1}\}\}).$$

When $d < \frac{1}{2}$, we have $T = \Omega(\max\{n^{d-1}, n^{d-1}\}) = \Omega(n^{d-1})$, which means ad hoc mode transmissions and infrastructure mode transmissions have the same impacts on the achievable throughput. When $\frac{1}{2} \le d < 1$, we have $T = \Omega(\max\{n^{d-1}, (n \log n)^{-\frac{1}{2}}\}) = \Omega(n^{d-1})$, i.e., infrastructure mode transmissions are more helpful to achieve higher throughput. Moreover, in both cases, we notice that the throughput capacity is only constrained by the number of destination nodes, i.e., n^d. Particularly, the achievable throughput is proportional to the number of destination nodes in the network, i.e.,

$$T = \Omega(n^{d-1}) = \Omega\left(\frac{n^d}{n}\right).$$

- Case 3: $w = \frac{1}{2}, 0 < d < 1, 0 < b < 1$.

In this case, the network area is a square, and the traffic is asymmetric. We obtain from (14) that

$$T = \Omega(\max\{\min\{n^{b-1}, n^{d-1}\}, \min\{(n \log n)^{-\frac{1}{2}}, n^{d-1}\}\}).$$

1. When $0 < d < \frac{1}{2}$, we have

$$T = \Omega(\max\{\min\{n^{b-1}, n^{d-1}\}, n^{d-1}\}),$$

i.e.,

$$T = \begin{cases} \Omega(\max\{n^{b-1}, n^{d-1}\}) = \Omega(n^{d-1}), \\ \qquad \text{when } 0 < b < d, \\ \Omega(\max\{n^{d-1}, n^{d-1}\}) = \Omega(n^{d-1}), \\ \qquad \text{when } 0 < d < b. \end{cases}$$

We find that when the number of base stations is smaller than the number of destination nodes, ad hoc mode transmissions can contribute more to the throughput capacity, while otherwise, ad hoc mode transmissions and infrastructure mode transmissions have the same impacts on the throughput capacity. Besides, no matter when $b < d$ or when $d < b$, the maximum achievable throughput is only constrained by the number of destination nodes. In other words, when the number of destination nodes is $O(\sqrt{n})$, we cannot improve the achievable throughput by putting more base stations in the network.

2. When $\frac{1}{2} \le d < 1$, we have

$$T = \Omega(\max\{\min\{n^{b-1}, n^{d-1}\}, (n \log n)^{-\frac{1}{2}}\}),$$

i.e.,

$$T = \begin{cases} \Omega(\max\{n^{b-1}, (n \log n)^{-\frac{1}{2}}\}), \\ \qquad \text{when } 0 < b < d, \\ \Omega(\max\{n^{d-1}, (n \log n)^{-\frac{1}{2}}\}) = \Omega(n^{d-1}), \\ \qquad \text{when } 0 < d < b. \end{cases}$$

We observe that in the case that $d \ge \frac{1}{2}$, the maximum achievable throughput is constrained by the number of base stations when $b < d$, and constrained by the number of destination nodes when $d < b$. Moreover, when $b < d$, infrastructure mode transmissions are more important in order to achieve higher throughput when $b \ge \frac{1}{2}$, and ad hoc mode transmissions are more important when $b < \frac{1}{2}$. When $d < b$, infrastructure mode transmissions are always more important.

- Case 4: $w < \frac{1}{2}, 0 < d < 1, 0 < b < 1$.

This is the most general case where the network area is a strip, the network traffic is asymmetric, and the number of base stations is between 1 and n. From (14), we notice that the achievable throughput contributed by infrastructure mode transmissions is constrained by the number of base stations and the number of destination nodes, while the achievable throughput contributed by ad hoc mode transmissions is constrained by the shape of network area and the number of destination nodes. Thus, the achievable throughput in hybrid wireless networks is related to all the three parameters, i.e., w, d, and b. Specifically, we obtain that

$$T = \begin{cases} \Omega\left(\max\left\{n^{b-1}, \frac{n^{w-1}}{\sqrt{\log n}}\right\}\right), \\ \qquad \text{when } d > b, \quad \text{and } d > w, \\ \Omega(\max\{n^{b-1}, n^{d-1}\}) = \Omega(n^{d-1}), \\ \qquad \text{when } d > b, \quad \text{and } d < w, \\ \Omega\left(\max\left\{n^{d-1}, \frac{n^{w-1}}{\sqrt{\log n}}\right\}\right) = \Omega(n^{d-1}), \\ \qquad \text{when } d < b, \quad \text{and } d > w, \\ \Omega(\max\{n^{d-1}, n^{d-1}\}) = \Omega(n^{d-1}), \\ \qquad \text{when } d < b, \quad \text{and } d < w. \end{cases} \tag{15}$$

Notice that only when $d > b$ and $d > w$, the maximum achievable throughput is determined by both the number of base stations and the shape of network area. In all the other cases, i.e., the maximum achievable throughput is only determined by the number of destination nodes. Moreover, in a special case $d \to 1, b \to 1$, and $w = \frac{1}{2}$, we have $T = \Omega(1)$.

5 Throughput Capacity with Randomly Placed Base Stations

In this section, we study the throughput capacity of hybrid wireless networks considering the m base stations are uniformly and independently placed. Notice that Lemmas 1 and 2 derived in Section 4 are still valid here.

Recall the definition of *Voronoi Tessellation*: given a set of m points in a plane, Voronoi tessellation divides the domain in a set of polygonal regions, the boundaries of which are the perpendicular bisectors of the lines joining the points. It has been shown in [12] (Lemma 4.1) that for every $\varepsilon > 0$, there is a Voronoi tessellation with the property that every Voronoi cell contains a disk of radius ε and is contained in a disk of radius 2ε. Then, for the m base stations in an extended network with area n, we can construct a Voronoi tessellation V_n for which

- (V1) Every Voronoi cell contains a disk of area $100n \log m/m$.
- (V2) Every Voronoi cell is contained in a disk of radius $2\rho(n)$, where $\rho(n) :=$ the radius of a disk of area $\frac{100n \log m}{m}$.

In this case, we consider each voronoi cell is a cell in the network.

Following the steps in Section 4, we first derive an achievable data rate when all the transmissions are carried out in infrastructure mode, denoted by T_i'.

Step 1: from source nodes to base stations.

The same as that in Section 4, we assume source nodes have low transmission power and they need to transmit packets to base stations via multiple hops.

Lemma 7. *In each Voronoi cell, there are at most $\frac{1{,}200n \log m}{m}$ nodes w.h.p.*

Proof. Consider Voronoi cell i. Let X_i be a random variable denoted as the number of nodes in the disk of radius $2\rho(n)$ containing cell i, and $E[X_i]$ the expectation of X_i. Then, we have $E[X_i] = \frac{400n \log m}{m}$. By Chernoff bounds, we obtain that

$$P\left(X_i > \frac{1{,}200n \log m}{m}\right) < e^{-\frac{400n \log m}{m} \cdot f(2)} = e^{-400 f(2) bn^{1-b} \log n},$$

where $f(2) = 3 \log 3 - 2 > 1$. Since $0 < b < 1$, the probability shown above approaches to 0 as $n \to \infty$.

Let Y_i be a random variable denoted as the number of nodes in Voronoi cell i. Then, we have $P(Y_i > \frac{1{,}200n \log m}{m}) \leq P(X_i > \frac{1{,}200n \log m}{m}) \to 0$ as $n \to \infty$. Thus, the probability that the number of nodes is at most $\frac{1{,}200n \log m}{m}$ in all Voronoi cells, denoted by $P(Y_i > \frac{1{,}200n \log m}{m} \forall i)$, can be calculated as

$$P\left(Y_i \leq \frac{1{,}200n \log m}{m} \forall i\right) \geq 1 - mP\left(Y_i > \frac{1{,}200n \log m}{m}\right) > 1 - n^b e^{-400 f(2) bn^{1-b} \log n},$$

which approaches to 1 as $n \to \infty$. □

Denote the throughput capacity in Step 1 by T_{i1}'. Since each node is a source node, along the line in Section 4.1, we can obtain that

$$T_{i1}' = \Omega\left(\frac{m}{n \log m}\right) = \Omega\left(\frac{n^{b-1}}{\log n}\right). \quad (16)$$

Step 2: Infrastructure Relay.

Denote the throughput capacity in Step 2 by T_{i2}'. We have

$$T_{i2}' = \Theta(1). \quad (17)$$

Step 3: from base stations to destination nodes.

Lemma 8. *In each Voronoi cell, w.h.p., there are at most $\frac{800n^d \log m}{m}$ destination nodes when $0 < b < d < 1$, and at most c_5, where $c_5 > \frac{1+b}{b-d}$, destination nodes when $0 < d < b < 1$.*

Proof. Consider Voronoi cell i. Let X_i be a random variable denoted as the number of destination nodes in the disk of radius $2\rho(n)$ containing cell i, and $E[X_i]$ the expectation of X_i. Then, we have $E[X_i] = \frac{400n^d \log m}{m}$. Following the steps in Lemma 4, we can obtain that when $b < d$,

$$P\left(X_i > \frac{800n^d \log m}{m}\right) < e^{-f(1) bn^{d-b} \log n} \to 0, \text{ as } n \to \infty,$$

where $f(1) = 2 \log 2 - 1 > 0$, and when $d < b$,

$$P(X_i > c_5) < \frac{e^{c_5}}{c_5^{c_5} \cdot (\frac{n^{b-d}}{\log n})^{c_5}} \to 0, \text{ as } n \to \infty,$$

where c_5 is a constant that will be determined later.

Let Y_i be a random variable denoted as the number of nodes in Voronoi cell i. Then, we have $P(Y_i > \frac{800n^d \log m}{m}) \leq P(X_i > \frac{800n^d \log m}{m}) \to 0$, as $n \to \infty$, when $b < d$, and $P(Y_i > c_5) \leq P(X_i > c_5) \to 0$, as $n \to \infty$, when $d < b$. Thus, similar to that in Lemma 4, we can obtain the results accordingly by choosing $c_5 > \frac{1+b}{b-d}$. □

We know from Lemma 5 that there are at most $2n^{1-d}$ source nodes that have the same destination node. Thus, in each Voronoi cell, w.h.p., the number of flows from base station to destination nodes is at most $\frac{800n^d \log m}{m} \times 2n^{1-d}$, i.e., $1{,}600bn^{1-b} \log n$, when $0 < b < d < 1$, and at most $c_5 \times 2n^{1-d}$, i.e., $2c_5 n^{1-d}$, when $0 < d < b < 1$.

Notice that when base stations are randomly placed, the shape of cells in the network are not regular any more, and hence we cannot assume the downlink transmission in Step 3 is one hop delivery as in Section 4.1. Instead, we assume here that base stations have limited transmission power and they cannot directly transmit to destination nodes within the Voronoi cells. This is in fact also the case in many Wireless Mesh Networks (WMNs) where the APs are weakly powered. In this part, we further assume base stations have the same transmission power as normal nodes. Thus, the packets are relayed from base stations to destination nodes via multiple hops. Notice that as shown in Fig. 3, in one Voronoi cell, a node has to relay packets for at most all the flows from the base station to destination nodes. Denote the throughput capacity in Step 3 by T_{i3}'. Then, we can obtain that

$$T_{i3}' = \begin{cases} \Omega\left(\frac{n^{b-1}}{\log n}\right), & \text{when } 0 < b < d < 1, \\ \Omega(n^{d-1}), & \text{when } 0 < d < b < 1. \end{cases} \quad (18)$$

Combining (16)-(18), we have

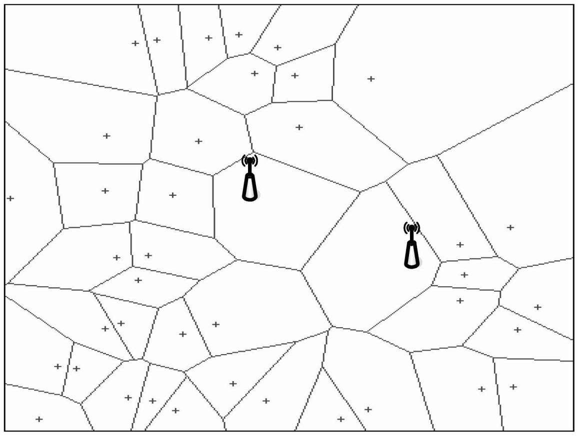

Fig. 3. A Voronoi tessellation example. The "+" symbols stand for the base stations, and we have replaced two points with base stations in the figure.

$$T_i' = \Omega(\min\{T_{i1}', T_{i2}', T_{i3}'\}) = \Omega\left(\min\left\{\frac{n^{b-1}}{\log n}, n^{d-1}\right\}\right). \quad (19)$$

Let T_a' denote an achievable data rate when all the transmissions are carried out in ad hoc mode. Notice that ad hoc mode transmissions do not rely on base stations. So T_a' is the same as that in (13), i.e.,

$$T_a' = \Omega\left(\min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right). \quad (20)$$

Denote as T' the throughput capacity of hybrid wireless networks when the base stations are uniformly and independently distributed. Similar to that in (14), we have

$$T' = \Omega\left(\max\left\{\min\left\{\frac{n^{b-1}}{\log n}, n^{d-1}\right\}, \min\left\{\frac{n^{w-1}}{\sqrt{\log n}}, n^{d-1}\right\}\right\}\right). \quad (21)$$

Note that $n^{d-1} = O(\frac{n^{b-1}}{\log n})$ when $d < b$, and $\frac{n^{b-1}}{\log n} = O(n^{d-1})$ when $b < d$. Thus, the analysis in Section 4.3 still holds here, and we can have a similar result to that in (15):

$$T' = \begin{cases} \Omega\left(\max\left\{\frac{n^{b-1}}{\log n}, \frac{n^{w-1}}{\sqrt{\log n}}\right\}\right), & \text{when } d > b, \text{ and } d > w, \\ \Omega\left(\max\left\{\frac{n^{b-1}}{\log n}, n^{d-1}\right\}\right) = \Omega(n^{d-1}), & \text{when } d > b, \text{ and } d < w, \\ \Omega\left(\max\left\{n^{d-1}, \frac{n^{w-1}}{\sqrt{\log n}}\right\}\right) = \Omega(n^{d-1}), & \text{when } d < b, \text{ and } d > w, \\ \Omega(\max\{n^{d-1}, n^{d-1}\}) = \Omega(n^{d-1}), & \text{when } d < b, \text{ and } d < w. \end{cases} \quad (22)$$

Specifically, only when $d > b$ and $d > w$, the maximum achievable throughput is determined by both the number of base stations and the shape of network area. In all the other cases, i.e., the maximum achievable throughput is only determined by the number of destination nodes. Moreover, the result in (22) is the same as that in (15) except for the case $d > b > w$, in which the random placement of base stations will cause a degradation factor of $\log n$ on the maximum achievable throughput compared to the regular placement.

In addition, we also notice that in a special case $d \to 1$, $b \to 1$, and $w = \frac{1}{2}$, we have $T' = \Omega(\frac{1}{\log n})$. In other words, in a square network with symmetric traffic, the achievable throughput can be $\Omega(\frac{1}{\log n})$ when the number of base stations is on the same order of the number of normal nodes, which is the same as that shown in [14].

6 Impacts of Power Propagation Model

Our analysis above employs the power propagation model in (1), which is widely used in the study of capacity of wireless networks such as [11], [12]. However, we notice that when d_{ij} in (1) is very small, the reception power level P_{ij} will be very high. Particularly, P_{ij} can be even higher than P_i, the transmission power, which is impossible. This problem was addressed in [2], [7], [9], [18] by upper bounding the reception power at each node, i.e.,

$$P_{ij} = C\frac{P_i}{(1 + d_{ij})^{\gamma}}. \quad (23)$$

In this section, we will show that the results in this paper still hold even with this new power propagation model.

By carefully checking the derivation process of our results, we find that the only issue that needs to be proved again with the new power propagation model is Lemma 2. In the proof of Lemma 2, considering the model in (23), we find that I_i', the cumulative interference at square s_i, changes from (4) into

$$\begin{aligned} I_i' &\le \sum_{j=1}^{J} 8j \times \frac{CP(n)}{[1 + (3j-2)l]^{\gamma}} \\ &\le \sum_{j=1}^{J} 8j \times \frac{CP(n)}{[(3j-2)l]^{\gamma}} \\ &= I_i \\ &\le \frac{8CP(n)}{l^{\gamma}} \cdot \frac{3\gamma - 5}{3\gamma - 6}. \end{aligned}$$

Besides, the reception power level at the receiver of s_i, denoted by R_i', changes from (5) into

$$R_i' \ge \frac{CP(n)}{(1 + \sqrt{5}l)^{\gamma}}.$$

Since $l = \sqrt{c \log n} > 1$, we have

$$R_i' \ge \frac{CP(n)}{(l + \sqrt{5}l)^{\gamma}} > \frac{CP(n)}{[(1 + \sqrt{5})l]^{\gamma}}.$$

Thus, by choosing $P(n) = c'' l^{\gamma}$ where $0 < c'' < +\infty$, the SINR at the receiver of s_i, denoted by $SINR_i'$, is

$$\begin{aligned} SINR_i' = \frac{R_i'}{N_0 + I_i'} &\ge \frac{\frac{CP(n)}{[(1+\sqrt{5})l]^{\gamma}}}{N_0 + \frac{8CP(n)}{l^{\gamma}} \cdot \frac{3\gamma-5}{3\gamma-6}} \\ &= \frac{c''C}{(1+\sqrt{5})^{\gamma}(N_0 + 8c''C\frac{3\gamma-5}{3\gamma-6})}, \end{aligned}$$

which is still a constant irrespective to the number of nodes n. As a result, Lemma 2 still holds under the new power propagation model, and hence our results obtained before also hold.

7 Conclusion

In this paper, we study the impacts of network topology and traffic pattern on the capacity of hybrid wireless networks. We have derived an achievable transmission rate when base stations are regularly and randomly distributed in the network, respectively. From these results, we can easily see that both network topology and traffic pattern have great impacts on network capacity. More interestingly, we observe that the number of destination nodes is more crucial to the network capacity. In both settings, we observe that only when $d > b$ and $d > w$, the maximum achievable throughput can be determined by both the number of base stations and the shape of network area. In all the other cases, the maximum achievable throughput is only constrained by the number of destination nodes. Moreover, the results in these two settings are the same except for the case $d > b > w$, in which the random placement of base stations will cause a degradation factor of $\log n$ on the maximum achievable throughput compared to the regular placement. Besides, we also show that our results actually hold for different power propagation models. Finally, notice that in this study we did not consider the special cases when $b = d$, and/or $d = w$ since the results in these special cases would be very straightforward following the analysis above.

Appendix A
Proof of Lemma 6

Consider an arbitrary square s_i in the network. Let N_{i1} and N_{i2} denote the number of source nodes which are located in squares with the same x-coordinate as s_i, and the number of destination nodes which are located in squares with the same y-coordinate as s_i, respectively. Referring to Section 4.2, we have shown that $E[N_{i1}] = n^w(c\log n)^{\frac{1}{2}}$ and $E[N_{i2}] = n^{d-w}(c\log n)^{\frac{1}{2}}$.

1. According to the Chernoff bound in (6), we obtain that

$$P(N_{i1} > 2n^w(c\log n)^{\frac{1}{2}}) < e^{-f(1)n^w(c\log n)^{\frac{1}{2}}},$$

where $f(1) = 2\log 2 - 1 > 0$. Since $0 < w \leq \frac{1}{2}$, as $n \to \infty$, we have $P(N_{i1} > 2n^w(c\log n)^{\frac{1}{2}}) \to 0$. Let $P(N_{i1} \leq 2n^w(c\log n)^{\frac{1}{2}}\forall i)$ denote the probability that for each square the number of source nodes located in squares with the same x-coordinate is at most $2n^w(c\log n)^{\frac{1}{2}}$. We can obtain that

$$\begin{aligned} P(N_{i1} &\leq 2n^w(c\log n)^{\frac{1}{2}})\forall i) \\ &\geq 1 - \frac{n}{c\log n}P(N_{i1} > 2n^w(c\log n)^{\frac{1}{2}}) \\ &> 1 - \frac{n}{c\log n}e^{-f(1)n^w(c\log n)^{\frac{1}{2}}}, \end{aligned}$$

which approaches to 1 as $n \to \infty$.

2. *First*, when $0 < w < d < 1$, we have

$$P(N_{i2} > 2n^{d-w}(c\log n)^{\frac{1}{2}}) < e^{-f(1)n^{d-w}(c\log n)^{\frac{1}{2}}},$$

which approaches to 0 as $n \to \infty$. Similar to that in 1, we can easily show that $P(N_{i2} \leq 2n^{d-w}(c\log n)^{\frac{1}{2}}\forall i) \to 1$ as $n \to \infty$.

Second, when $0 < d < w < \frac{1}{2}$, according to the Chernoff bound in (6), we can obtain that

$$\begin{aligned} P(N_{i2} > (1+\delta)E[N_{i2}]) &< e^{-E[N_{i2}][(1+\delta)\log(1+\delta)-\delta]} \\ &= \frac{e^{\delta E[N_{i2}]}}{(1+\delta)^{(1+\delta)E[N_{i2}]}}. \end{aligned}$$

Let $1+\delta = \frac{c_4}{E[N_{i2}]} = c_4 n^{w-d}(c\log n)^{-\frac{1}{2}}$ where c_4 is a constant that will be determined later. Then, we have

$$\begin{aligned} P(N_{i2} > c_4) &< \frac{e^{[c_4 n^{w-d}(c\log n)^{-\frac{1}{2}}-1]n^{d-w}(c\log n)^{\frac{1}{2}}}}{[c_4 n^{w-d}(c\log n)^{-\frac{1}{2}}]^{c_4}} \\ &= \frac{e^{c_4 - n^{d-w}(c\log n)^{\frac{1}{2}}}}{c_4^{c_4}[n^{w-d}(c\log n)^{-\frac{1}{2}}]^{c_4}} \\ &< \frac{e^{c_4}}{c_4^{c_4}} \cdot [n^{d-w}(c\log n)^{\frac{1}{2}}]^{c_4}, \end{aligned}$$

which approaches to 0 as $n \to \infty$. Besides, we can also obtain that

$$\begin{aligned} P(N_{i2} \leq c_4\forall i) &\geq 1 - \frac{n}{c\log n}P(N_{i2} > c_4) \\ &> 1 - \frac{e^{c_4}}{c_4^{c_4}} \cdot n^{(d-w)c_4+1} \cdot (c\log n)^{\frac{1}{2}c_4 - 1}. \end{aligned}$$

When we choose $c_4 > \frac{2}{w-d}$, we can get $(d-w)c_4 + 1 < -1$, and hence $P(N_{i2} \leq c_4\forall i) \to 1$ as $n \to \infty$.

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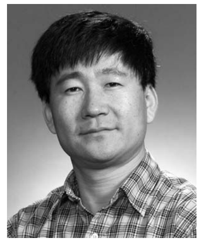

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